

CHAPTER 25

CAPACITANCE

A brief mention of capacitors as a means of storing electrical charge was made in Chapter 8. It was stated that any two pieces of metal separated from each other by an insulator will have the ability to store electrical energy. The advantage of storing energy in a capacitor, as against storing energy in a magnetic field, is that the energy can be retained in a capacitor after the voltage applied to the capacitor has been removed. In the inductor, the energy is dissipated as soon as the current in the inductor ceases. Factors which determine the capacitance and the quantity of charge in a capacitor will be studied in this Chapter.

25.1 FACTORS THAT DETERMINE CAPACITANCE

If two parallel metal plates are separated from each other by an insulator and connected to a d.c. supply, electrons will move from the negative side of the supply and disperse over the plate connected to the negative terminal of the supply. These electrons on the negative plate will, by the laws of electrostatic charge, repel electrons from the opposite plate, leaving this plate with a positive charge. (Figure 25.1).

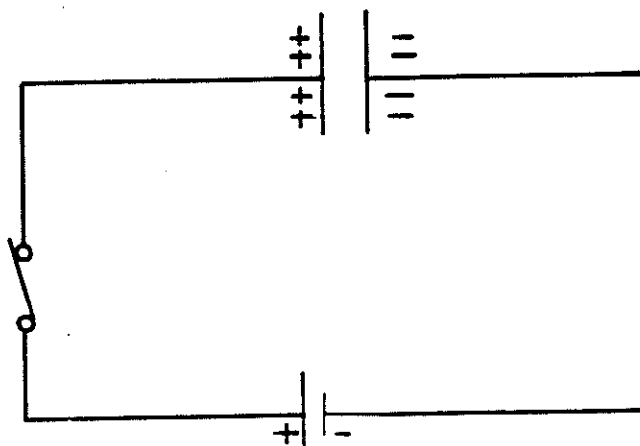


Figure 25.1

If the source voltage 'E' is removed from the capacitor, the charge will remain on the plates. (Figure 25.2).

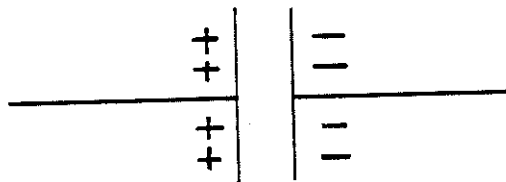


Figure 25.2

The capacitor is said to store charge or electrical energy. The ability to store charge is due to the electrostatic effect of the plates and the insulated area (called a dielectric) between the plates. The capacitance of a capacitor depends on three factors —

1. The area of the plates (A). Increase in plate area increases the capacitance of a capacitor.
2. The distance between the plates (d). Increase in the distance between the plates results in decreased capacitance. The further away the positive plate is from the negative plate, the lower the influence of the negative charge on that plate.
3. The type of dielectric (K) between the plate. Air is a dielectric and is often used as such in a variable capacitor. In fixed capacitors, the air is replaced by a dielectric which increases the charge that can be stored in the capacitor. 'K' is known as the dielectric constant.

The dielectric constant consists of two parts, the permittivity of free space (ϵ_0) and the relative permittivity (ϵ_r) of the dielectric. The permittivity of free space is 8.85×10^{-12} farads per meter. The relative permittivity of a dielectric compares its permittivity with that of air, which is given a figure of unity (1). Typical relative permeabilities of materials used as dielectrics in capacitors is given in Table 25.1.

Table 25.1

MATERIAL	RELATIVE PERMITTIVITY
Air	1
Transformer Oil	4
Plastic	7
Glass	6
Mica	5
Paraffin Paper	2.5

Combining the factors that govern the capacitance of a capacitor gives —

$$C = \frac{KA}{d} \text{ farads}$$

or $C = \frac{\epsilon_0 \epsilon_r A}{d} \text{ farads}$

where ϵ_0 = permittivity of free space

ϵ_r = relative permittivity of the dielectric

C = capacitance in farads (F)

K = dielectric constant

A = cross-sectional area in square metres

d = distance between the plates in metres

Example 25.1

A parallel plate capacitor has plates whose surface areas are both 2500 millimetres square, and are separated by a distance of 5 millimetres. Calculate the capacitance of the capacitor if the dielectric has a relative permittivity of 8.

$$\epsilon_0 = 8.85 \times 10^{-12}$$

$$\epsilon_r = 8$$

$$A = 2500 \times 10^{-6} \text{ m}$$

$$d = 5 \times 10^{-3} \text{ m}$$

$$C = ? \text{ F}$$

$$C = \frac{\epsilon_0 \epsilon_r A}{d}$$

$$= \frac{8.85 \times 10^{-12} \times 8 \times 2500 \times 10^{-6}}{5 \times 10^{-3}}$$

$$= 35.4 \times 10^{-12} \text{ F}$$

The farad is rarely used in practical capacitors as the unit is too large. Most capacitors are in the microfarad range. A more practical answer for example 25.1 would be —

$$C = 35.4 \times 10^{-6} \mu\text{F}$$

25.2 CHARGE ON A CAPACITOR

In Chapter 4 electric charge was found to be the product of current and time.

$$Q = It$$

This expression applied only where the current is a constant value. The charge (Q) in a capacitor is the product of the applied e.m.f. and the capacitance of the capacitor.

$$Q = CE$$

Where —

Q = charge in coulombs

C = capacitance in farads

E = e.m.f. applied to the capacitor in volts

As the microfarad is normally used in practice the equation for charge becomes —

$$Q = CE \times 10^{-6} \text{ coulombs}$$

Example 25.2

A 30 microfarad parallel plate capacitor is charged from a 500 volt d.c. supply. Calculate the charge on the capacitor.

$$\begin{array}{ll} E = 500 \text{ V} & Q = CE \times 10^{-6} \\ C = 30 \text{ uF} & = 30 \times 500 \times 10^{-6} \\ Q = ? \text{ coulombs} & = 0.015 \text{ coulombs} \end{array}$$

An increase in the e.m.f. applied to a capacitor will result in an increase in the charge stored by a capacitor.

25.3 TYPES OF CAPACITORS

Capacitors are known by the material that is used as their dielectric. Typical dielectrics in use and their permittivities are listed in table 25.1. The appearance and construction of capacitors vary greatly. Figure 25.2 shows a range of capacitors used in the electrical industry.

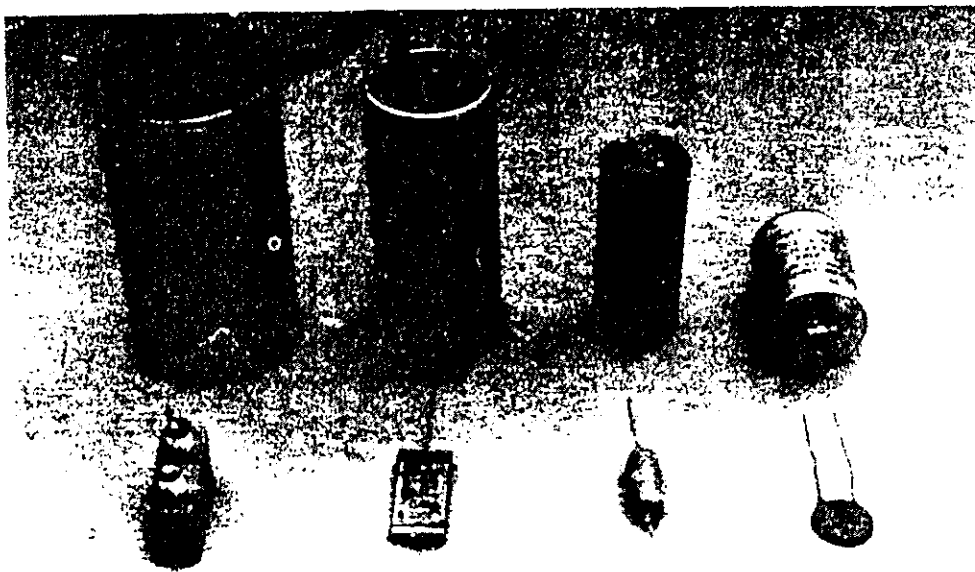


Figure 25.2

The largest capacitor, which is a parafin paper capacitor, is used to improve the performance of single phase motors. The capacitors decrease in physical size to the smallest one which is typical of the type used in electronic circuit. The capacitors in figure 25.2 are fixed value capacitors. Variable capacitors which use air as their dielectric, are available in a wide number of sizes and shapes. (Figure 25:3).

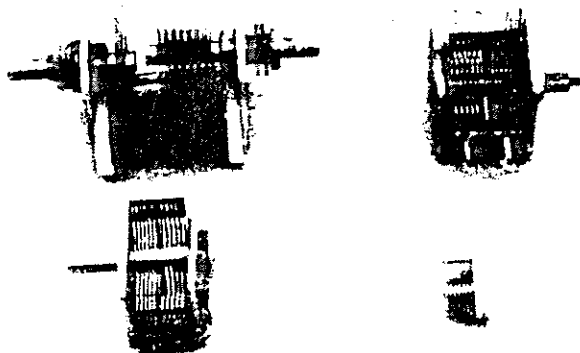


Figure 25.3

Another form of capacitor is the electrolytic capacitor. As typical electrolytic capacitor has aluminium foil electrodes and a dielectric of borax paste. The capacitor is 'polarised', that is, it has definite positive and negative connections. These are indicated by a '+' or '-' sign, by arrows which point in the correct direction of current through the capacitor, or by colouring the positive end of the capacitor. (Figure 25.4)



Figure 25.4

An electrolytic capacitor may be quickly destroyed if it is connected in reverse polarity. Electrolytic capacitors have a very high ratio of charge to capacitor volume. As an example the capacitor in figure 25.5 is approximately half full scale but has a capacitance of $1000 \mu\text{F}$. Electrolytic capacitors, due to their high charge-volume ratio, are used extensively in electronic circuitry.

25.4 CHARGING AND DISCHARGING CAPACITORS

When a d.c. supply is connected to a capacitor which is completely discharged, there will be a rush of electrons from the negative side of the source. The abundant electrons on the negative plate of the source will attempt to distribute themselves between the source and the capacitor plate. The movement of electrons to the plate of the capacitor will force an equal number of electrons away from the opposite plate of the capacitor. Electrons only move in the circuit external to the capacitor, not across the dielectric. The electrons in the dielectric may have their orbits altered, but they do not pass from plate to plate. (Figure 25.5).

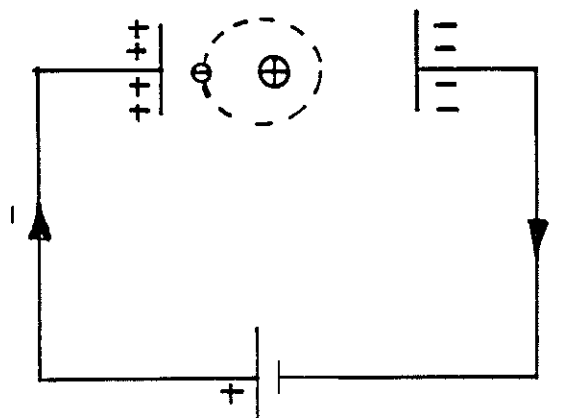


Figure 25.5

After the initial rush of electrons from the source to the capacitor plate, the difference between the charge on the source and that on the plate will be decreased. Fewer electrons will move from the source to the capacitor. When the plate reaches the same state of charge as the negative side of the source, electron movement will cease. During this process electrons have been repelled from the positive plate in the same number as were received by the negative plate. The current in the external circuit of the capacitor was a maximum at the instant the capacitor had been connected to the supply, and has gradually decreased to zero. (Figure 25.6).

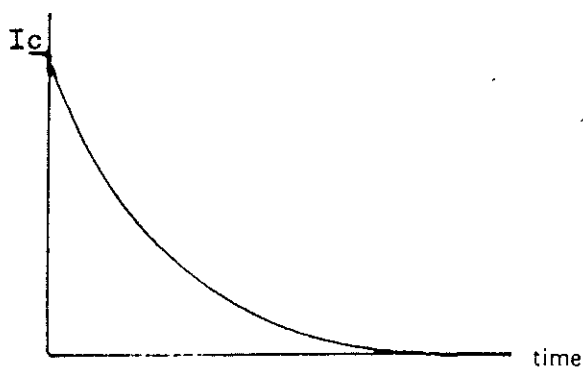


Figure 25.6

The potential difference across the capacitor is zero at the instant of connecting the capacitor to the supply, and has increased to the value of the supply voltage. (Figure 25.7)

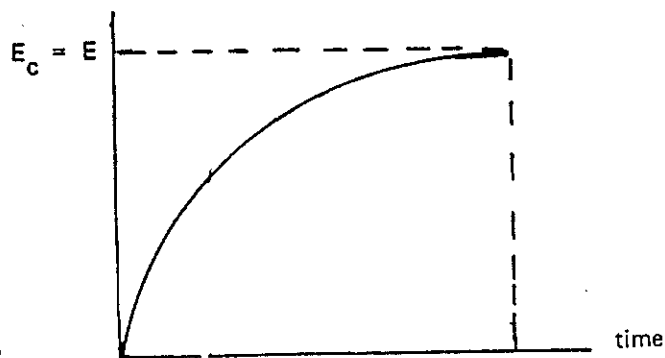


Figure 25.7

When the capacitor is disconnected from the supply and is discharging into a load, the current and capacitor voltage will be a maximum at the instant it is connected to the load and then decreased to zero. A graph of the decreasing current and voltage would be similar to that shown in figure 25.6.

25.5 TIME CONSTANT OF AN R.C. CIRCUIT

As with inductance, it is impossible to have a purely capacitive circuit. Even if a purely capacitive circuit was possible, the connection of an uncharged capacitor to a d.c. supply could cause damage to the supply because an uncharged capacitor acts as a short circuit (maximum current). Most capacitive d.c. circuit besides having the resistance of the connecting wires, have a resistor included in the circuit to limit the maximum current drawn from the supply. Conversely, resistance is included in the circuit of a discharging capacitor, as a short circuit across a charged capacitor may cause serious damage to the capacitor.

The capacitor voltage and current that occur when a discharged capacitor is connected to a d.c. supply are shown in Figure 25.8.

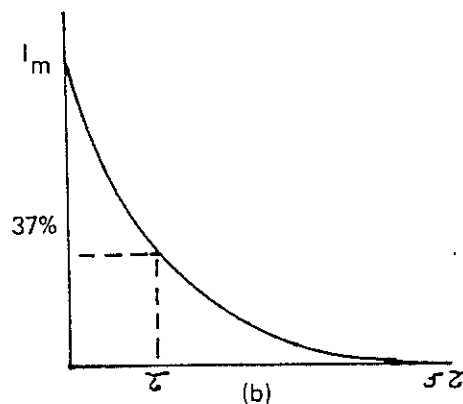
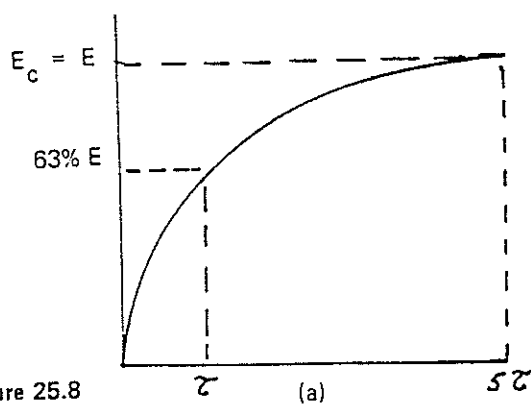


Figure 25.8

When the capacitor is discharged the capacitor voltage and current decrease similar to the charging current in Figure 25.8(b).

The rate at which current and voltage vary is dependant on the values of capacitance and resistance in the circuit, or the time constant of the circuit. In a resistive/capacitive d.c. circuit the time constant (τ) is equal to the product of the resistance and the capacitance in the circuit and is again 63% of the initial value.

$$\tau = RC \text{ seconds}$$

Example 25.3

A series circuit connected to a 500 volt d.c. supply consists of a resistance of $100\text{ k}\Omega$ and capacitor of 10 microfarads. Calculate —

- (a) the time constant of the circuit.
- (b) the voltage drop across the capacitor after the capacitor has been charged for one time constant.

$$R = 100\text{ k ohm}$$

$$C = 10 \times 10^{-6}\text{ F}$$

$$E = 500\text{ V}$$

$$\tau = ?\text{ seconds}$$

$$E_c = ?\text{ V}$$

$$\tau = RC$$

$$= 100 \times 10^3 \times 10 \times 10^{-6}$$

$$= 1\text{ second}$$

$$E_c = 63\%E$$

$$= \frac{63}{100} \times 500$$

$$= 315\text{ V}$$

TUTORIALS 1.25

- (1) Calculate the capacitance of a capacitor whose dielectric has a relative permeability of 5, a plate area of 3 metres squared and a plate separation of 0.1 millimetres.
- (2) Determine the charge on a $100\mu\text{F}$ capacitor connected to a 200 volt d.c. supply.
- (3) When a capacitor is charged from a 250 volt d.c. supply it stores a charge of 0.1 coulombs. Calculate the capacitance of the capacitor.
- (4) Determine the time constant of a circuit consisting of a $1000\mu\text{F}$ capacitor and an $8.2\text{ k}\Omega$ resistor.
- (5) A series circuit connected to a 500 volt d.c. supply consists of a $4.7\text{ M}\Omega$ resistor and a $25\mu\text{F}$ capacitor. Calculate:-
 - (a) Time constant of the circuit
 - (b) The capacitor voltage after one time constant
 - (c) The capacitor voltage after five time constants
 - (d) The time taken for the capacitor voltage to equal the supply voltage.